

# Performance of a Planar Filter Using a 0° Feed Structure

Chih-Ming Tsai, *Member, IEEE*, Sheng-Yuan Lee, *Student Member, IEEE*, and Chin-Chuan Tsai, *Student Member, IEEE*

**Abstract**—The advantage of using a 0° feed structure in filter design is that two extra transmission zeros are created in the stopband while the passband response remains unchanged. This feed structure is analyzed by using transmission matrices. A new lumped-circuit model for a coupled resonator filter is then proposed to take into account the effects of this feed structure. Finally, the feed structure is applied to the design of a cross-coupled filter. All the theoretical analysis and design procedures have been successfully verified by experiment results.

**Index Terms**—Coupled transmission lines, distributed parameter filters, microwave circuits, transmission-line resonators.

## I. INTRODUCTION

IT IS WELL known that there are two suitable feed points on a half-wavelength open transmission-line resonator for a given loaded  $Q$  [1]. For a direct-coupled hairpin filter, these feed points are shown in Fig. 1(a). Similarly they are also found in a cross-coupled miniaturized hairpin filter, as shown in Fig. 1(b). Usually feed points (i) and (ii) are used because the others are not accessible or they are close to other resonators and may result in the increase of unwanted coupling.

However, it is interesting to find that filters designed with different tapped-line feed points could be significantly different in their frequency responses. For example, a non-0° feed structure, as shown in Fig. 2(a), is usually used in the design of a four-pole cross-coupled filter [2]. However, it has been found recently that by using a 0° feed structure, as shown in Fig. 2(b), two transmission zeros near the passband are created and the stopband rejection is significantly increased [3]. In this paper, the characteristics of a 0° feed structure are further explored by a theoretical analysis. Based on this work, the applications of this feed structure are also extended to the other types of filters such as cross-coupled filters.

At the beginning of this paper, a 0° feed structure is analyzed by using transmission matrices. The equations for the zero positions are derived. The lumped-circuit model for a coupled resonator filter is then modified to take into account the effects of different feed points. Two filters with different feed structures were designed to verify the theoretical analysis and to present one of the applications.

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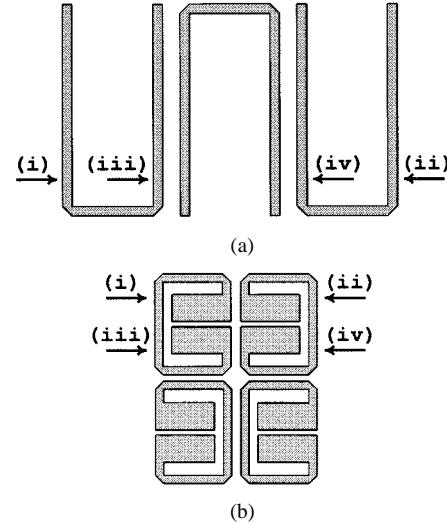


Fig. 1. Possible feed points of: (a) a direct-coupled hairpin filter and (b) a cross-coupled miniaturized hairpin filter.

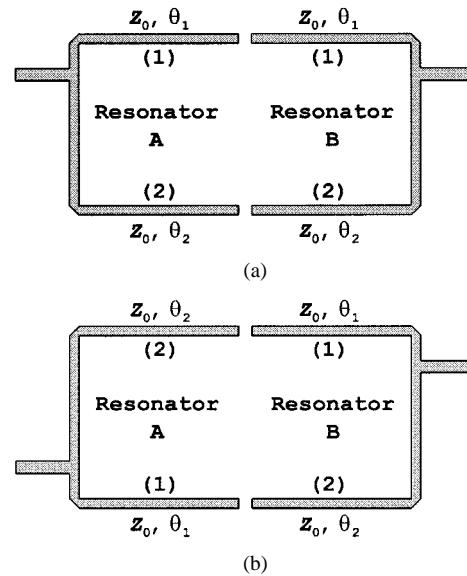


Fig. 2. (a) Non-0° feed structure. (b) 0° feed structure.

Also, the circuit model for four-pole cross-coupled filters is modified to include the effects of a 0° feed structure. The experiment results of a four-pole cross-coupled filter with a 0° feed structure are given and compared with those of the corresponding filter in [3]. The theoretical work and design rules in this paper can be extended to other higher order filters.

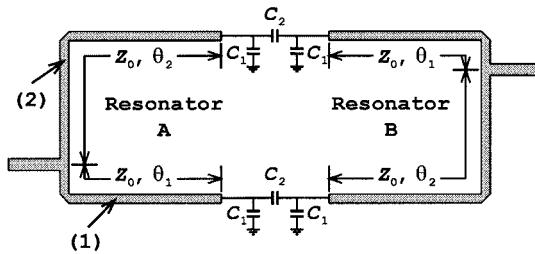


Fig. 3. Equivalent circuit and its parameters for a 0° feed structure.

## II. ANALYSIS OF DIFFERENT FEED STRUCTURE

Coupled resonators are building blocks of a filter. There are several types of coupling structures, including electric (capacitive) coupling, magnetic (inductive) coupling, and mixed coupling structures. Fig. 2 shows one type of electric coupling structures. Two half-wavelength transmission-line resonators are arranged as a ring with two gaps. This circuit and similar ones are usually used in four-pole cross-coupled filters, such as shown in Fig. 1(b). Two possible feed structures are shown in Fig. 2. The electrical delays of the upper and lower paths of the coupling structure in Fig. 2(a) are not the same at its fundamental resonant frequency. This kind of symmetric feed structure is commonly used and, in this paper, is referred to as a non-0° feed structure. The other possible feed structure, as shown in Fig. 2(b), was recently proposed in [3]. It is skew symmetric and is called a 0° feed structure because the difference between the electrical delays of the upper path and the lower path is 0°.

In Fig. 3, the coupling gaps between the two resonators in Fig. 2(b) are modeled as  $\pi$ -networks, where the values of  $C_1$  and  $C_2$  could be found from [4]. For a lower microwave frequency range, the value of  $\omega C_1$  is usually small ( $C_1 < 0.01$  pF) and will be neglected in the following analysis.

The transmission matrices of the lower and upper signal paths in a 0° feed structure are found in (1) and (2), shown at the bottom of this page. With the aid of circuit theory, the transmission matrix of the whole circuit can be written as (3), shown

at the bottom of this page. From (1) and (2), it is clear that  $A_u + A_l = D_u + D_l$ ,  $B_u = B_l$ , and  $C_u = C_l$ . The transmission matrix of a 0° feed structure can then be simplified as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{A_u + A_l}{2} & \frac{B_u}{2} \\ \frac{(A_u + A_l)^2 - 4}{2B_u} & \frac{A_u + A_l}{2} \end{bmatrix}. \quad (4)$$

By using network-parameter conversion, the transmission coefficient can be found as

$$S_{21} = \frac{4B_u Z_L}{B_u^2 + 2(A_u + A_l)B_u Z_L + [(A_u + A_l)^2 - 4]Z_L^2} \quad (5)$$

where the parameter  $Z_L$  is the system impedance (usually 50  $\Omega$ ).

Since  $B_u$  is finite, the necessary and sufficient condition for the existence of the transmission zeros is  $B_u = 0$  and the denominator of  $S_{21}$  is not equal to zero. If  $B_u = 0$ , it is found that

$$\tan \theta_1 + \tan \theta_2 = 1/Z_0 \omega C_2. \quad (6)$$

For a general filter design (i.e., 2%  $\leq$  fractional bandwidth  $\leq$  15%) in a lower microwave frequency range,  $C_2$  is usually at or smaller than the order of 0.1 pF, and  $\theta_1$  and  $\theta_2$  are generally designed as  $1.2\theta_1 < \theta_2 < 1.8\theta_1$  for the desired loaded  $Q$ . Therefore, (6) can be further reduced to either

$$\tan \theta_1 \approx 1/Z_0 \omega C_2 \quad (7)$$

or

$$\tan \theta_2 \approx 1/Z_0 \omega C_2. \quad (8)$$

Since  $C_2$  is small, it also means that the transmission zeros will be at the frequencies when  $\theta_1 \approx \pi/2$  or  $\theta_2 \approx \pi/2$ . Under the condition  $B_u = 0$ ,  $(A_u + A_l)^2 - 4 = (\cos \theta_1 / \cos \theta_2 + \cos \theta_2 / \cos \theta_1)^2 - 4$  cannot be zero. Therefore, transmission zeros are created because  $\lim_{B_u \rightarrow 0} S_{21} = 0$ .

$$\begin{bmatrix} A_l & B_l \\ C_l & D_l \end{bmatrix} = \begin{bmatrix} \cos(\theta_1 + \theta_2) + \frac{Y_0}{\omega C_2} \cos \theta_1 \sin \theta_2 & jZ_0 \sin(\theta_1 + \theta_2) - j \frac{\cos \theta_1 \cos \theta_2}{\omega C_2} \\ jY_0 \sin(\theta_1 + \theta_2) + j \frac{Y_0^2}{\omega C_2} \sin \theta_1 \sin \theta_2 & \cos(\theta_1 + \theta_2) + \frac{Y_0}{\omega C_2} \sin \theta_1 \cos \theta_2 \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} A_u & B_u \\ C_u & D_u \end{bmatrix} = \begin{bmatrix} \cos(\theta_1 + \theta_2) + \frac{Y_0}{\omega C_2} \sin \theta_1 \cos \theta_2 & jZ_0 \sin(\theta_1 + \theta_2) - j \frac{\cos \theta_1 \cos \theta_2}{\omega C_2} \\ jY_0 \sin(\theta_1 + \theta_2) + j \frac{Y_0^2}{\omega C_2} \sin \theta_1 \sin \theta_2 & \cos(\theta_1 + \theta_2) + \frac{Y_0}{\omega C_2} \cos \theta_1 \sin \theta_2 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{A_u B_l + A_l B_u}{B_u + B_l} & \frac{B_u B_l}{B_u + B_l} \\ \frac{[A_u B_l + A_l B_u][B_u D_l + B_l D_u] - (B_u + B_l)^2}{(B_u + B_l) B_u B_l} & \frac{B_u D_l + B_l D_u}{B_u + B_l} \end{bmatrix} \quad (3)$$

In the passband, i.e., when  $\theta_1 + \theta_2 \approx \pi$ , the transmission matrix can be calculated from (4) as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -1 & j \frac{\cos^2 \theta_1}{2\omega C_2} \\ 0 & -1 \end{bmatrix}. \quad (9)$$

The transmission coefficient can then be found as

$$S_{21} \approx \frac{-1}{2 - j \cos^2 \theta_1 / 2\omega C_2 Z_L}. \quad (10)$$

The analysis of a  $0^\circ$  feed structure is now complete. Similar analysis could be applied to a non- $0^\circ$  feed structure. The transmission matrices for the lower and upper signal paths of a non- $0^\circ$  feed structure are

$$\begin{bmatrix} A_l & B_l \\ C_l & D_l \end{bmatrix} = \begin{bmatrix} \cos 2\theta_2 + \frac{Y_0}{\omega C_2} \sin \theta_2 \cos \theta_2 & j Z_0 \sin 2\theta_2 - j \frac{\cos^2 \theta_2}{\omega C_2} \\ j Y_0 \sin 2\theta_2 + j \frac{Y_0^2}{\omega C_2} \sin^2 \theta_2 & \cos 2\theta_2 + \frac{Y_0}{\omega C_2} \sin \theta_2 \cos \theta_2 \end{bmatrix} \quad (11)$$

and

$$\begin{bmatrix} A_u & B_u \\ C_u & D_u \end{bmatrix} = \begin{bmatrix} \cos 2\theta_1 + \frac{Y_0}{\omega C_2} \sin \theta_1 \cos \theta_1 & j Z_0 \sin 2\theta_1 - j \frac{\cos^2 \theta_1}{\omega C_2} \\ j Y_0 \sin 2\theta_1 + j \frac{Y_0^2}{\omega C_2} \sin^2 \theta_1 & \cos 2\theta_1 + \frac{Y_0}{\omega C_2} \sin \theta_1 \cos \theta_1 \end{bmatrix} \quad (12)$$

where, from (11) and (12), it is clear that  $A_l = D_l$  and  $A_u = D_u$ . The transmission matrix of a non- $0^\circ$  feed structure can then be simplified as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{A_u B_l + A_l B_u}{B_u + B_l} & \frac{B_u B_l}{B_u + B_l} \\ \frac{[A_u B_l + A_l B_u]^2 - (B_u + B_l)^2}{(B_u + B_l) B_u B_l} & \frac{A_u B_l + A_l B_u}{B_u + B_l} \end{bmatrix}. \quad (13)$$

The transmission coefficient can be found in (14), shown at the bottom of the following page.

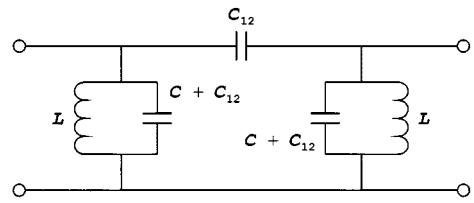
Since  $B_u$  and  $B_l$  are finite, the transmission zero may be created if either one of the conditions: 1)  $B_u = 0$ ; 2)  $B_l = 0$ ; or 3)  $B_u = B_l = 0$  is satisfied. However, 3) means  $\theta_1 = \theta_2$  and is not possible for a general filter design. If 1) or 2) is fulfilled, the following equations can be derived, respectively:

$$\tan \theta_1 = 1/2Z_0\omega C_2 \quad (15)$$

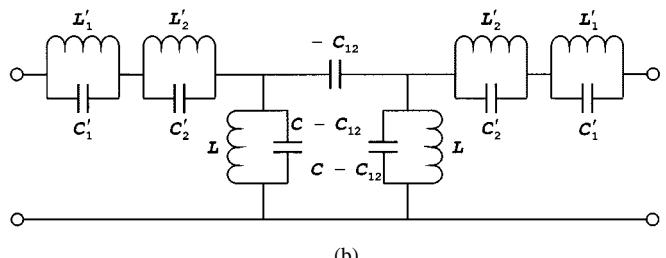
or

$$\tan \theta_2 = 1/2Z_0\omega C_2. \quad (16)$$

Unfortunately, under each circumstance, the value of  $(A_u B_l + A_l B_u)^2 - (B_u + B_l)^2$  is equal to zero. It can be shown that



(a)



(b)

Fig. 4. Lumped-circuit models for coupled resonator circuit with: (a) a non- $0^\circ$  feed structure and (b) a  $0^\circ$  feed structure.

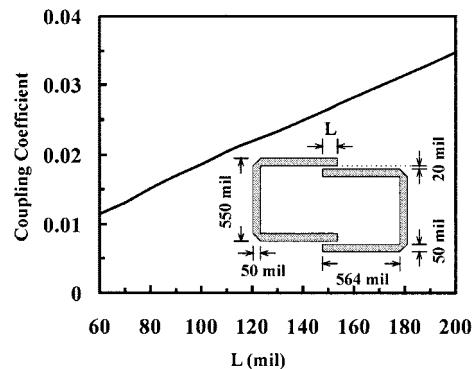
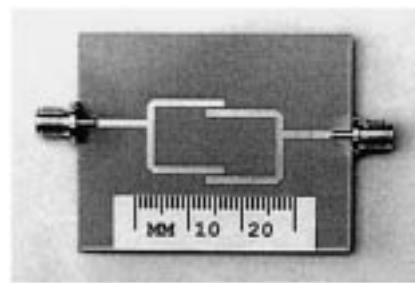
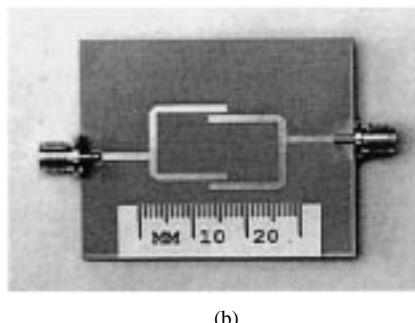


Fig. 5. Design curve for the coupling coefficient.



(a)



(b)

Fig. 6. Second-order filters with: (a) a non- $0^\circ$  feed structure and (b) a  $0^\circ$  feed structure.

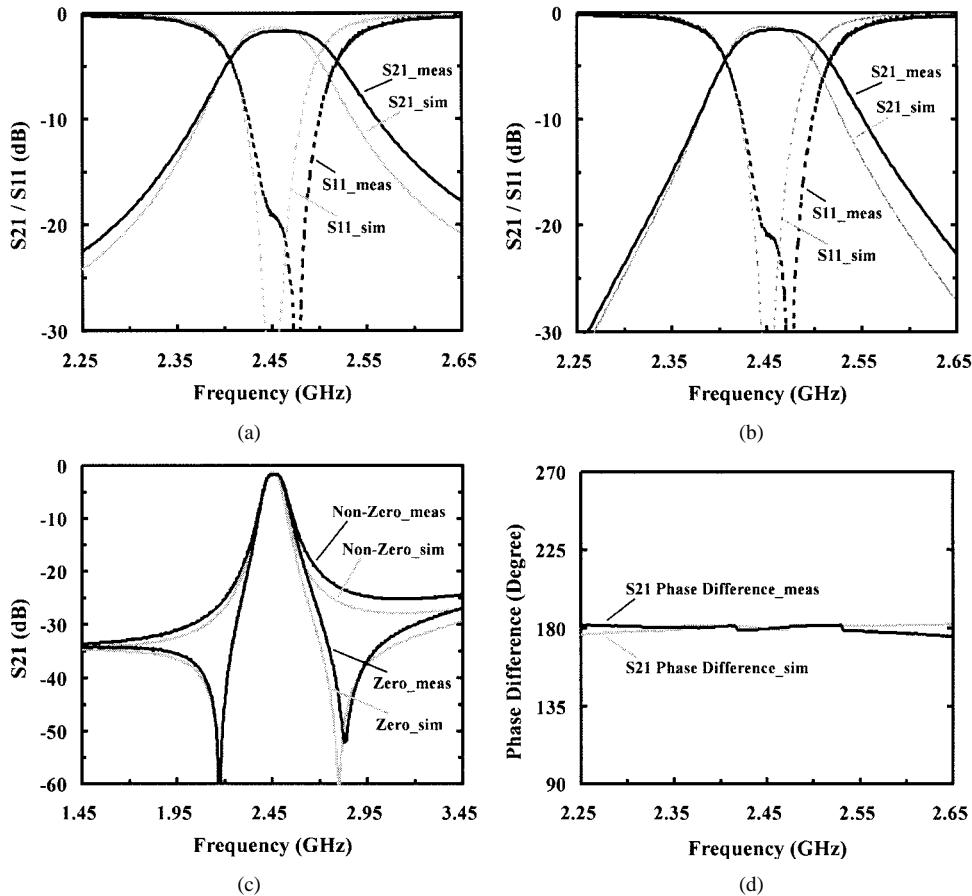


Fig. 7. Passband responses of filters with: (a) the non-0° feed structure and (b) the 0° feed structure. (c) Comparison of the stopband responses of these filters. (d) The difference of  $S_{21}$  phase response of these filters.

$\lim_{B_u \rightarrow 0} S_{21}$  or  $\lim_{B_l \rightarrow 0} S_{21}$  in (14) in each case has a finite value and the transmission zero cannot be created.

In the passband, the transmission matrix of the non-0° feed structure is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -j \frac{\cos^2 \theta_1}{2\omega C_2} \\ 0 & 1 \end{bmatrix}. \quad (17)$$

The transmission coefficient is found as

$$S_{21} \approx \frac{1}{2 - j \cos^2 \theta_1 / 2\omega C_2 Z_L} \quad (18)$$

which is negative of the one of a 0° feed structure.

Therefore, it can be concluded that two extra transmission zeros can be created if a 0° feed structure is used. The passband transmission responses of filters with different feed structures have only a 180° phase difference.

The model usually used for a coupled resonator circuit with a non-0° feed structure is shown in Fig. 4(a). Based upon the previous discussions, the model for a coupled resonator circuit with a

0° feed structure is proposed and shown in Fig. 4(b). Two parallel resonators in a series arm are added to each side of the model for the transmission zeros and to keep the model symmetric. One resonator is resonant at the frequency when  $\theta_1$  approaches  $\pi/2$  and the other is resonant at the frequency when  $\theta_2$  is approximately  $\pi/2$ . These two zeros are close to and on the opposite sides of the passband and, hence, significantly increase the stopband rejection. Moreover, the sign of the coupling coefficient between the resonators is changed. This model could be applied to other similar coupling structures with miniaturized hairpin resonators or stepped-impedance hairpin resonators in [3].

The transmission zeros can also be explained by field analysis. In Fig. 2(a), the transmission-line section (1) of resonator A forms a resonant tank when its length is about a quarter-wavelength. The transmission-line section (1) of resonator B is resonant at the same frequency. The field coupling between these two resonant tanks is strong. Therefore, no transmission zeros appear. A similar thing happens when transmission-line section (2) resonates. On the other hand, for a 0° feed structure in Fig. 2(b), the two resonant tanks are not directly coupled to each

$$S_{21} = \frac{2B_u B_l (B_u + B_l) Z_L}{B_u^2 B_l^2 + 2(A_u B_l + A_l B_u) B_u B_l Z_L + [(A_u B_l + A_l B_u)^2 - (B_u + B_l)^2] Z_L^2} \quad (14)$$

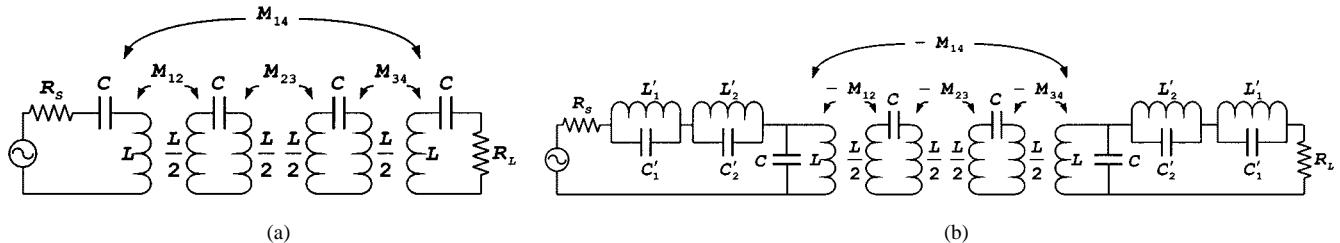


Fig. 8. Lumped-circuit models for four-pole cross-coupled filters with: (a) a non- $0^\circ$  feed structure and (b) a  $0^\circ$  feed structure.

other. The signal coupled from resonators A to B is minimized and, therefore, transmission zeros could be found.

### III. BUTTERWORTH FILTERS WITH DIFFERENT FEED STRUCTURES

Two second-order Butterworth filters with different feed structures were designed to verify the discussions in Section II. Each filter was designed to have the center frequency at 2.45 GHz and 3% bandwidth. The normalized loaded  $Q$  and the coupling coefficient for this type of filter can be found from [5] as  $q_1 = q_2 = 1.414$  and  $k_{12} = 0.707$ . The actual loaded  $Q$  and the coupling coefficient  $K_{12}$  can be calculated from  $Q_1 = Q_2 = q_1/BW$  and  $K_{12} = BW \cdot k_{12}$ , where  $BW$  is the percentage bandwidth. For this example,  $Q_1$  and  $K_{12}$  were found to be 47.1 and 0.021.

These two filters were fabricated on Rogers RO3003 substrates, with a relative dielectric constant of 3.00, a loss tangent of 0.0013, and a thickness of 20 mil. In order to increase the coupling coefficient, coupled-line structures were used. The coupling coefficient as a function of the coupled-line length is calculated using Agilent Momentum and is shown in Fig. 5. The design of the feed points to yield the desired loaded  $Q$  can be found in [1], [6] as  $\theta_1 = 100.5^\circ$  and  $\theta_2 = 79.5^\circ$  at the center frequency 2.45 GHz. From Section II, the two zeros should be at 2.19 and 2.77 GHz, when  $\theta_1$  or  $\theta_2$  approaches  $\pi/2$ , respectively.

The photographs of these two filters are shown in Fig. 6(a) and (b). The simulation results and measured data are given in Fig. 7. Fig. 7(a) and (b) shows the passband responses. The center frequencies of both circuits are slightly shifted to 2.46 GHz. The passband insertion loss is approximately 1.6 dB, and the return loss is greater than 15 dB for both filters. The comparison of stopband responses is shown in Fig. 7(c). It is clear that the filter with a  $0^\circ$  feed structure has two extra transmission zeros. From the measured data, one zero is at 2.18 GHz and the other is at 2.83 GHz, which are in agreement with the simulations. This is also well predicated by the above theoretical analysis with an error less than 2%. These zeros are close to the passband and make the stopband rejection much better than the filter with a non- $0^\circ$  feed structure. The difference of  $S_{21}$  phase response of filters with different feed structures is shown in Fig. 7(d). It shows that the transmission responses of these two filters have a  $180^\circ$  phase difference in the passband, which agrees with the discussion in Section II.

### IV. CROSS-COUPLED FILTER WITH A $0^\circ$ FEED STRUCTURE

Cross-coupled filters, which can realize an elliptic or a quasi-elliptic response, are good for increasing selectivity [7],

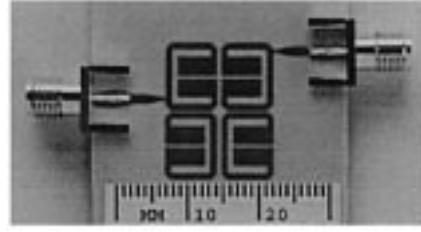
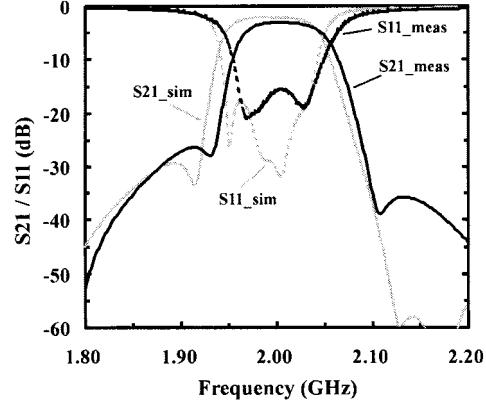
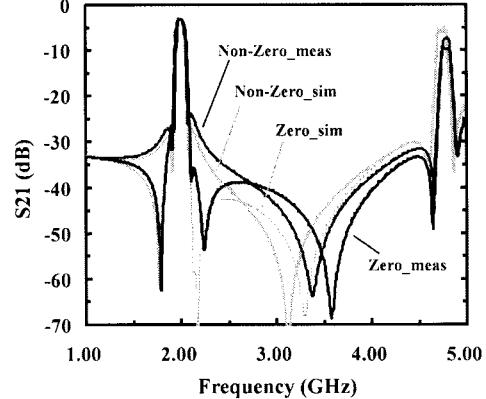


Fig. 9. Cross-coupled filter with a  $0^\circ$  feed structure.



(a)



(b)

Fig. 10. (a) Passband response of the cross-coupled filter with a  $0^\circ$  feed structure. (b) Stopband responses of the filters with a  $0^\circ$  feed structure and a non- $0^\circ$  feed structure.

[8]. This is because they have finite-frequency zeros near the passband and, therefore, the response skirts are steeper. However, this type of filters has some disadvantages. One of them is the possible degradation of stopband rejection, especially for four-pole cross-coupled planar filters, as shown in Fig. 1(b). To achieve the passband response, their input and output sections

are close. Therefore, the stopband rejection degrades due to the direct coupling from the input resonator to the output resonator.

In a previous paper [3], a cross-coupled filter with miniaturized hairpin resonators, which has an extra tunable transmission zero in the stopband, was proposed to reduce this effect. Based on the discussion in Section II, it is clear that a 0° feed structure can also be used for solving this problem. The two extra transmission zeros near the passband can be helpful in increasing the stopband rejection. A lumped-circuit model for an ordinary four-pole cross-coupled filter was proposed in [9] and is shown in Fig. 8(a). If the feed structure is changed from a non-0° one to a 0° alternative, all the signs of coupling coefficients must be changed and two parallel resonators must be added in series, both to the input and output for symmetry, as shown in Fig. 8(b).

The four-pole cross-coupled filter designed by using the miniaturized hairpin resonators in [3] was modified. The feed structure was replaced by a 0° alternative. A photograph of the new filter is shown in Fig. 9. Fig. 10(a) shows the passband experiment results. The insertion loss is compared with the one of the original design with a non-0° feed structure in [3], and both are shown in Fig. 10(b). It is clear that the passband response is not changed, but the stopband rejection is significantly increased because of the two zeros created by a 0° feed structure.

## V. CONCLUSIONS

A lumped-circuit model for electric coupling structures with a 0° feed structure has been proposed. It has been shown that this type of circuit and other circuits using an ordinary non-0° feed structure have the same passband response, except a 180° phase difference. The advantage of using a 0° feed structure is that two extra transmission zeros are created at the frequencies close to and on the opposites of the passband. The selectivity and stopband rejection of this type of filters are, therefore, significantly increased. Two second-order filters with different feed structures have been designed to verify these discussions. Besides, a circuit model for cross-coupled filters has also been proposed to take into account the effects of a 0° feed structure. The increase of selectivity has been proven by experiments. The tuning of these transmission zeros can be achieved by using impedance transformers and it has been reported in [6].

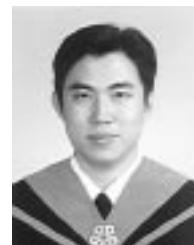
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